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**454 Assignment 1**

**1)**Prove the correctness of the Algorithm Maximum on page 30 in Chapter 0 of the courseware.

Maximum correctness:

Prove that ⩝n ≥ 1, when the program terminates:

max = a|∃*t,* 1≤*t* ≤n, where ⩝*u*, 1≤*u* ≤n, L[*t* ] ≥L[*u*] and L[*t* ] *=* a

(*induction basis*)When n=1, L contains 1 element

Note: on line 1, max is set to L[1], thus *t*=1

Control never enters for loop because in line 2, *i* is set to 2 and 2>n (∵n=1)

Algorithm terminates and max = L[*t* =1], and n =1 (1≤*t* ≤n)

The only *u* ∈ [1,n] is 1=*t* thus L[*u* ] = L[*t* ] ⩝*u*, 1≤*u* ≤n

hence, max = a|∃*t,* 1≤*t* ≤n, where ⩝*u*, 1≤*u* ≤n, L[*t* ] ≥L[*u*] and L[*t* ] *=* a

(*inductive hypothesis*)

Suppose for m1<n;

max = a|∃*t,* 1≤*t* ≤n, where ⩝*u*, 1≤*u* ≤n, L[*t* ] ≥L[*u*] and L[*t* ] *=* a

(*inductive step*)

Consider input of size n ≥ 2

Initially, control reaches line 1 and max is set to L[1], and

at line 2 control enters the loop, *i* is set to 2

When control reaches line 2 for the time,

control remains in the loop since *i* =m=n≤n.

max = L[*t*], 1≤*t* <*i*-1*,* ⩝*u*, 1≤*u* ≤*i*-1, L[*t* ] ≥L[*u*] (Inductive Hypothesis)

Control reaches line 3, max is set to L[*i* ] if max < L[i], otherwise max is not changed

Thus. after line 3 executes, max = L[*t*], 1≤*t* <*i,* ⩝*u*, 1≤*u* ≤*i*, L[*t* ] ≥L[*u*]

When control reaches line 2 for the time, control exist the loop since *i*=(m+1)≥n

Thus, when control reaches line 2 for the time;

max = a|∃*t,* 1≤*t* ≤n, where ⩝*u*, 1≤*u* ≤n, L[*t* ] ≥L[*u*] and L[*t* ] *=* a

■

**2)**Prove that

Assume , prove

(by assumption)

(definition of )

(by assumption)

(definition of o)

(UI on c and EI on )

(EI on c, c’, and )

(∵ )

(∵ )

(transitivity of ≤)

(∵ ⩝)

(UG on andEG on )

(definition of o)

■

**3)** Prove or Disprove: For every c ∈ and every ⋃{0}, (cn) ∈ Θ()

Disprove by counterexample:

let c = 0.5 and (n) = , therefore, (cn) =

**=**

=

=

=

= (Where 0 < ℰ = < 1)

=

=

= 0

Therefore, by Theorem 0.2(d) ∈ o() (cn) ∈ o((n))

Furthermore, by theorem 0.1(c) (cn)∉ Θ((n))

■

**4)a)** Prove ∈ o():

Thus, by Theorem 0.2(d), ∈ o()

**b)** Prove ∈ Θ(lglg(n!)):

let c = 1, n0 = 4, Thus;

lgn ∈ O(lglg(n!))

()

let c = 2, n0 = 2, Thus;

lglg(n!) ∈ O(lgn)

Therefore, ∈ Θ(lglg(n!)), be Theorem 0.3(b)

**c)** Prove lglg(n!) ∈ o():

Thus, by Theorem 0.2(d), n ∈ o()

∈ Θ(lglg(n!)) (By 4(b) of this assignment)

∈ Θ(lgn) (By Theorem 0.5(b))

Therefor, by Question 2 of this assignment, where:

= lglg(n!), , and ,

lglg(n!) ∈ o()

**d)** Prove ∈ o():

=

=

=

= (L’Hopital’s Rule)

= (Simplify)

= (L’Hopital’s Rule)

= (Simplify)

= (L’Hopital’s Rule)

= (Simplify)

= (L’Hopital’s Rule)

= (Simplify)

=

**Note:** lg10 < 4, therefor > 0

=

=0

Therefor, by Theorem 0.2(d), ∈ o()

**e)** Prove ∈ o((n2 – 3n)3 ):

=

=

= 0

Therefor, by Theorem 0.2(d), ∈ o((n2 – 3n)3 )

**f)** Prove (n2 – 3n)3 ∈ o():



ii)

iii)

iv)

v)

Thus, by Limit Chain Rule with and , and

by Limit Chain Rule with and , and

by Limit Chain Rule with and , and

by Limit Chain Rule with and .

= 0

Thus, by Theorem 0.2(d), (n2 – 3n)3 ∈ o()

**g)** Prove ∈ o()

Note: ⩝ n > 1, lgn > 0

Therefore, by Example 3 on page 48, ∈ o()

**Therefore**, the functions in ascending asymptotic order are:

■

**5)** Solve the Following Relations

**a)** *T* (n) = *T* () + n

Since *T* () = *T* (), we have = 1, =, and (n) = m

0 < 1 1 = for some

Therefor, (n) = n = (n1) = for some

Moreover, for sufficiently large n,

a(n/b) = 1(n/(10/9)) = n/(10/9)

= n

≤ cn, where 0 < c = < 1

Thus, by Case 3, we have *T* (n) = (n)

**b)** *T*(n) = 3*T* () + nlgn

Here, = 3, = 2, and (n) = nlgn

= for some

(L’Hopital’s Rule)

= (Simplify)

= (L’Hopital’s Rule)

= (Simplify)

= (Rearrange)

=

= 0

Thus, by Theorem 0.2(a) n⋅lgn ∈ () for some

Therefore, by Case 1, T(n) =

**c)** *T*(n) = 4*T* () + n2 + n

Here, = 4, = 2, and (n) = n2 + n

= n2

**lemma 5c.1:**

= (L’Hopital’s Rule)

= (L’Hopital’s Rule)

= 4 > 0

Therefore, by Theorem 0.2(c), n2 + n (n2)

(n) = n2 + n = (n2) (lemma 5c.1)

= (n2⋅1)

= (⋅lg0n)

**=**(⋅lgkn) (k=0)

Thus, by Case 2, *T* (n) = (⋅lgn)